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CHARACTERISTICS OF RADIATIVE HEAT TRANSFER IN MULTIZONE SYSTEMS TAKING ACCOUNT OF ISOTROPIC SCATTERING

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A procedure for calculating heat-transfer characteristics in absorbing and isotropically scattering media is developed and tested by applying it to internally fired furnaces.

Optimization of the thermal performance of industrial heating units imposes increasingly rigid requirements on the accuracy and refinement of heat-transfer calculations. Taking account of radiation scattering by solid particles suspended in streams of dusty gases in working chambers of internally fired furnaces and fireboxes may significantly affect the calculated results. Therefore, it is of practical interest, particularly since detailed information has appeared [1-3] on the radiation characteristics of dusty streams and luminous flames, to present a solution within the framework of the zonal method of the heat-transfer problem, taking account of radiation scattering in complex three-dimensional systems filled with a radiating and absorbing medium.

The available literature ([4-10] et al.) contains reports on the effect of scattering on radiative transfer for such relatively simple radiating systems as a slab, an isothermal medium with uniform radiation characteristics, black walls, etc. In [11] a procedure was developed and heat-transfer calculations were performed for a steel-making furnace, taking account of radiation scattering by the Monte Carlo method. This solution was based on the determination of photon mean free paths by using random numbers. Generalized angular coefficients of radiation between zones ψ_{ij} were calculated taking account of scattering in a dusty gas medium [11]. In the calculation of the reduced resolving radiation coefficients f_{ij} the reradiation of energy by surface zones was determined from the linear radiation equations [12-14]. The temperatures of volume and surface zones were calculated by solving the system of nonlinear algebraic heat-transfer and heat-balance equations of the zones [15, 16].

We have developed a procedure for determining the generalized angular and resolving radiation coefficients and have solved a zonal heat-transfer problem in the working chamber of a steel-making furnace, taking account of isotropic scattering of radiation by solving the system of linear algebraic equations of radiative heat transfer. The use of an isotropic scattering indicatrix for real media is justified to a certain degree by the fact that under certain conditions, in particular for relatively large dust particles, the scattered part of the radiation flux can be assumed isotropic, and the diffracted part extremely elongated forward, that is, coincident with the transmitted radiation [17, 18].

The essence of the proposed method consists in the following. First the generalized angular radiation coefficients ψ_{ij}^{att} are found by the Monte Carlo method [15], or by some other method if the radiating systems are simple, by replacing the absorption coefficient α of the volume zones by the attenuation coefficient $K = \alpha + \beta$, where β takes account of isotropic scattering only; the diffracted radiation is taken into account in the transmitted radiation. The values of the coefficients ψ_{ij}^{att} obtained in this way differ from the generalized angular coefficients ψ_{ij}, which take account of scattering by the method used in [11], since the part of the energy scattered into volume zone j is included in the value of the coefficient ψ_{ij}^{att}. An analysis of the radiative transfer equation in an absorbing and scattering medium [19]

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TABLE 1. Values of Generalized Angular Coefficients ψ_{ij} and Reduced Resolving Coefficients of Radiation f_{ij} Calculated by Proposed Method (I) and by Monte Carlo Method (II). The Wall (23rd surface zone) Radiates*

No. of absorbing zone		No. of calculated part	ψ_{ij}(I)	ψ_{ij}(II)	f_{ij}(I)	f_{ij}(II)
Volume zones	1	I	0,001	0,001	0,001	0,001
	2		0,000	0,000	0,000	0,000
	3	II	0,002	0,002	0,004	0,004
	4		0,000	0,000	0,000	0,000
	5	III	0,007	0,005	0,009	0,008
	6		0,004	0,002	0,003	0,003
	7	IV	0,015	0,008	0,013	0,013
	8		0,016	0,009	0,012	0,012
	9	V	0,054	0,029	0,036	0,037
	10		0,079	0,042	0,051	0,051
Surface zones	11	I	0,001	0,002	0,002	0,003
	12		0,002	0,001	0,002	0,002
	13		0,000	0,000	0,000	0,000
	14	II	0,005	0,006	0,006	0,006
	15		0,001	0,002	0,004	0,003
	16		0,000	0,000	0,001	0,001
	17	III	0,014	0,018	0,020	0,020
	18		0,008	0,005	0,014	0,011
	19		0,004	0,006	0,007	0,007
	20	IV	0,045	0,053	0,065	0,064
	21		0,049	0,058	0,066	0,069
	22		0,048	0,052	0,044	0,043
	23	V	0,123	0,148	0,177	0,179
	24		0,265	0,282	0,275	0,276
	25		0,257	0,269	0,188	0,186

*1, 3, 5, 7, 9, fuel spray zones; 2, 4, 6, 8, 10, zones of diathermic and absorbing layers; 11, 14, 17, 20, 23, wall zone; 12, 15, 18, 21, 24, roof zones; 13, 16, 19, 22, 25, bath zones.

$$\frac{dI}{dx} = -\alpha I - \beta I \tag{1}$$

shows that over a finite path length l the ratio of the scattered energy to the sum of the energies scattered and absorbed is equal to the Schuster number Sc. *

Over a length l the sum of the scattered and absorbed energy flux densities is

$$I_{\text{att}} = I_0\{1 - \exp[-(\alpha + \beta)l]\}; \tag{2}$$

the transmitted energy flux density at a distance x from the origin of the ray is

$$I = I_0 \exp[-(\alpha + \beta)x], \tag{3}$$

and the energy flux density scattered in a path length dx at a distance x from the origin of the ray, as follows from (1) and (3), is

$$dI_{\text{sc}} = \beta I_0 \exp[-(\alpha + \beta)x]\,dx. \tag{4}$$

Integrating over a length l, we obtain

$$I_{\text{sc}} = \beta I_0 \int_0^l \exp[-(\alpha + \beta)x]\,dx$$

$$= \frac{\beta}{\alpha + \beta} I_0\{1 - \exp[-(\alpha + \beta)l]\}. \tag{5}$$

By using Eq. (2)

*A. S. Nevskii was consulted in this part of the work.

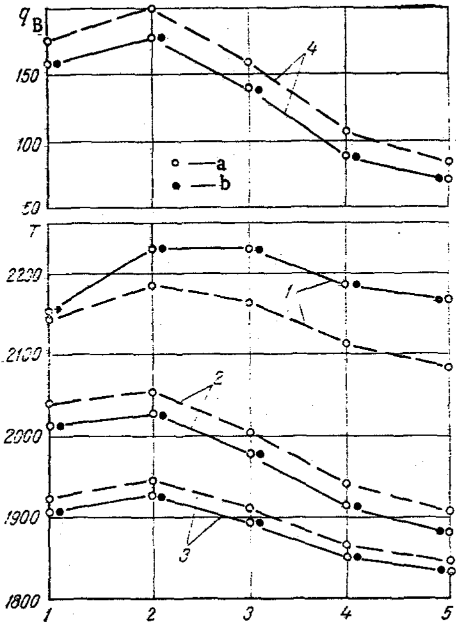

Fig. 1. Temperature distribution T in °K of fuel spray (1), roof surface (2), slag (3), and heat absorption density of bath q_B (4), in kW/m^2 along length of furnace calculated by neglecting scattering (dashed lines) and by taking scattering into account (solid lines) by the Monte Carlo method [11] (a) and by the proposed method (b). The number of calculated parts is plotted along the axis of abscissas.

$$\frac{I_{sc}}{I_{att}} = \frac{\beta}{\alpha + \beta} = \text{Sc}; \tag{6}$$

i. e., the fraction of the energy scattered in each volume zone is equal to the Schuster number in that zone, independently of the level of values of α and β, the amount of radiant energy entering the zone, and the ray length in the zone.

Thus, by determining the attenuation of the energy in zone j it is possible to judge the amount of radiation scattered and absorbed in that zone. The fraction of the radiant energy radiated from zone i which is scattered in volume zone j, taking account of the energy reaching zone j by replacing the absorption coefficients in the volume zones by attenuation coefficients, can be determined from the generalized angular coefficient ψ_{ij}^{att} as $(\text{Sc})_j \varphi_{ij}^{att}$.

Assuming that the scattering in each of m_1 volume zones is uniform and isotropic, i. e., the character of the scattered and self-radiation in these zones is the same, the expressions for the generalized angular radiation coefficients ψ_{ij} taking account of scattering can be written in the following form:

for volume absorbing zone j,

$$\psi_{ij} = \psi_{ij}^{att}[1 - (\text{Sc})_j] + \sum_{k=1}^{m_1} (\text{Sc})_k \psi_{ik}^{att} \psi_{kj} \tag{7}$$

$$(i = 1, 2, \ldots, n + m; \quad j = 1, 2, \ldots, m);$$

for surface absorbing zone j,

$$\psi_{ij} = \psi_{ij}^{att} + \sum_{k=1}^{m_1} (\text{Sc})_k \psi_{ik}^{att} \psi_{kj} \tag{8}$$

$$(i = 1, 2, \ldots, n + m; \quad j = 1, 2, \ldots, n).$$

Substituting Eqs. (7) and (8) into the expressions for the reduced resolving coefficients f_{ij}, and taking account of reradiation by surface zones [14], we obtain for the volume absorption of zone j

$$f_{ij} = \psi_{ij} + \sum_{p=1}^{n_1} R_p \psi_{ip} f_{pj} = \psi_{ij}^{att}[1-(Sc)_j]$$

$$+ \sum_{k=1}^{m_1} (Sc)_k \psi_{ik}^{att} \psi_{kj} + \sum_{p=1}^{n_1} R_p \left[\psi_{ip}^{att} + \sum_{k=1}^{m_1} (Sc)_k \psi_{ik}^{att} \psi_{kp} \right] f_{pj}$$

$$= \psi_{ij}^{att}[1-(Sc)_j] + \sum_{p=1}^{n_1} R_p \psi_{ip}^{att} f_{pj}$$

$$+ \sum_{k=1}^{m_1} (Sc)_k \psi_{ik}^{att} \left(\psi_{kj} + \sum_{p=1}^{n_1} R_p \psi_{kp} f_{pj} \right) \qquad (9)$$

$$(i = 1,\ 2,\ \ldots,\ n+m;\quad j = 1,\ 2,\ \ldots,\ m).$$

Taking account of the fact that for the volume absorption of zone j

$$f_{kj} = \psi_{kj} + \sum_{p=1}^{n_1} R_p \psi_{kp} f_{pj}, \qquad (10)$$

Eq. (9) can be written as

$$f_{ij} = \psi_{ij}^{att}[1-(Sc)_j] + \sum_{p=1}^{n_1} R_p \psi_{ip}^{att} f_{pj} + \sum_{k=1}^{m_1} (Sc)_k \psi_{ik}^{att} f_{kj} \qquad (11)$$

$$(i = 1,\ 2,\ \ldots,\ n+m;\quad j = 1,\ 2,\ \ldots,\ m).$$

An expression for the reduced resolving coefficients f_{ij} can be obtained similarly by taking account of isotropic scattering of radiation in volume zones and reradiation by surface zones when the absorbing zone j is a surface zone:

$$f_{ij} = \psi_{ij}^{att} A_j + \sum_{p=1}^{n_1} R_p \psi_{ip}^{att} f_{pj} + \sum_{k=1}^{m_1} (Sc)_k \psi_{ik}^{att} f_{kj} \qquad (12)$$

$$(i = 1,\ 2,\ \ldots,\ n+m;\quad j = 1,\ 2,\ \ldots,\ n).$$

It should be noted that if the radiating system has $(n-n_1)$ surface zones with nondiffuse reflection and $(m-m_1)$ volume zones with anisotropic scattering, ψ_{ij}^{att} is calculated by the Monte Carlo method [11], taking account of the appropriate laws of reflection and the values of the scattering indicatrix in these zones.

In contrast with the method of calculating reduced resolving coefficients used in [11], which takes account of scattering by the Monte Carlo method, the present method makes it possible to calculate generalized angular coefficients by simple computational algorithms without taking account of scattering and with smaller expenditures of machine time. The generality of the results obtained can be increased by using the proposed method, since the generalized angular radiation coefficients ψ_{ij}^{att} can be used in subsequent calculations to determine the reduced resolving coefficients of radiation in systems with different values of the Sc number but the same values of the attenuation coefficients for the volume zones. The smaller number of pseudorandom numbers required to calculate the generalized angular radiation coefficients without taking account of scattering increases the accuracy of the results obtained, since the requirements for the number of pseudorandom numbers are not always related to the exacting requirements for the uniformity and dynamics of the distribution of these numbers on the interval (0, 1) [20].

In addition, for relatively small zones the use of the Monte Carlo method to calculate generalized angular coefficients by taking account of radiation scattering leads to a decrease in accuracy, since the number of individual radiant streams transmitted which can be tracked, and, consequently, also the number scattered in a volume zone, is appreciably smaller than the number of individual radiant streams initially directed from the radiating zone. In this case the error in calculating the distribution of scattered energy increases in comparison with the error in calculating the distribution of energy among zones as a result of direct radiation. The error in calculating the distribution of scattered energy by using the solution of the system of linear algebraic equations (7) and (8) or (11) and (12) remains at the level of the error in determining the generalized angular radiation coefficients.

On the basis of our method we developed an algorithm and universal operating programs for a Minsk-22 computer to calculate the reduced resolving radiation coefficients f_{ij}, taking account of scattering in the volume zones and reradiation by the surface zones, and the coefficients of radiative exchange a_{ij} for the zonal nonlinear algebraic heat-transfer and heat-balance equations [15, 16] for a number of zones $(n + m) \leq 52$. The program provides for the simultaneous checking of the accuracy of the calculations of ψ_{ij}, f_{ij}, and a_{ij}, based on the property of closure (e.g., $\sum_{j=1}^{n+m} \psi_{ij} = 1$ and $\sum_{j=1}^{n+m} f_{ij} = 1$) and the correction (increase in accuracy of the calculation) of a_{ij} based on reciprocity; i.e., $a'_{ij} = a'_{ji} = (a_{ji} + a_{ij})/2$, where a_{ij} and a_{ji} are coefficients of radiative exchange between zones i and j, calculated directly from f_{ij} by using the equations of [15], and a'_{ij} and a'_{ji} are the refined coefficients of radiative exchange. The meaning of the refinement is that a'_{ij} and a'_{ji} are calculated by using results obtained in a number of trials equal to the sum of the numbers of trials used to obtain a_{ij} and a_{ji}.

Using the proposed method we investigated heat transfer in a steel-making furnace. The working chamber of the furnace was divided into five parts for the calculation, corresponding to the five charging ports. Each part consisted of two volume zones (a fuel spray in the lower part of the working chamber and air or combustion products in the upper part) and three surface zones (wall, roof, and bath). The total number of zones was 25. The emissivity of the furnace lining was taken as 0.8 and that of the bath as 0.6. The basic version was heat transfer in a purely absorbing (nonscattering) medium, i.e., one with an attenuation coefficient K equal to the absorption coefficient ($K = \alpha$; $\beta = 0$). The distribution of absorption coefficients over the volume zones and other characteristics of the heat-transfer model for this version are given in [14, 16].

To compare with the basic version, variants were computed in which the attenuation of radiation due both to absorption and scattering was taken into account. The Sc number in these variants was arbitrarily taken as 0.5; i.e., $\alpha = \beta$. The attenuation coefficients in the volume zones were equal to the attenuation coefficients for the basic version. The generalized angular coefficients, the reduced resolving coefficients of radiation, and the temperature distributions were calculated both by the method described in the present paper and by the method of [11].

A comparison of the calculated values of the radiation coefficients ψ_{ij} and f_{ij} for the zonal model of a steel-making furnace by the two methods shows that in spite of a significant change in the calculation scheme the radiation coefficients f_{ij} which take account of scattering in the volume zones are negligibly different. The absolute difference in the values of f_{ij} for rows does not exceed 0.008 on the average. The maximum difference for the whole f_{ij} matrix is 0.020, and for individual columns of f_{ij} does not exceed 0.003 (Table 1).

Analysis of the temperature distribution of the fuel spray, roof, and slag, and the density of heat absorption by the bath along the length of the furnace for the basic version and the variants which include scattering (Fig. 1) showed that decreasing the value of the absorption coefficient and simultaneously taking account of scattering leads to a significant change in the heat-transfer pattern in the working chamber. This shows the necessity of separating the experimental values of the attenuation coefficient into absorption and scattering components for heat-transfer calculations.

The temperatures of the volume and surface zones and the total heat absorbed by the bath calculated by taking account of scattering in solving the heat-transfer problem in a steel-making furnace by the Monte Carlo method [11] and by the proposed method do not differ by more than 0.13%.

Thus, taking account of radiation scattering may in individual cases lead to a significant correction of the heat-transfer pattern in industrial heating units. The proposed method of calculating radiative heat-transfer characteristics by taking account of isotropic scattering gives an accuracy which is adequate for engineering calculations.

NOTATION

α, β, K, absorption, scattering, and attenuation coefficients of medium, m^{-1}; ψ_{ij}, average generalized angular coefficient of radiation from zone i to zone j; f_{ij}, reduced resolving coefficient of radiation from zone i to zone j; a_{ij}, coefficient of radiative exchange between zones i and j, $W/°K^4$; x, l, variable and constant ray lengths, m; I_0, I, initial and running values of radiation flux density, W/m^2; Sc, Schuster number; n, m, numbers of surface and volume zones in system; n_1, number of surface zones with diffuse reflection; m_1, number of volume zones with isotropic scattering; R_p, reflectivity of surface zone p; A_j, absorptivity (emissivity) of surface zone j.

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SCATTER OF SPECIFIC HEAT AND DENSITY OF FILLED MATERIALS IN SAMPLES OF FINITE DIMENSION

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The probability of obtaining values of the specific heat and density of filled samples that differ from the mean is considered.

In experimental determinations of the specific heat and density of small samples, it is necessary to estimate the possible deviation of the properties of the given sample from the mean values. Such a deviation may occur because, when the sample is small, the number of filler particles that the considered volume contains is a random variable, and hence characteristics such as the specific heat and density of the sample are also random variables.

Let c be the specific heat of the considered filled material. For simplicity, we shall consider a composite of no more than two materials. Since

$$c = c_1 P_{1\,\mathrm{gr}} + c_2 P_{2\,\mathrm{gr}} \tag{1}$$

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